

Cyclic Magnetic Universe

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Recent works have shown the important role Nonlinear Electrodynamics (NLED) can have in two crucial questions of Cosmology, concerning particular moments of its evolution for very large and for low-curvature regimes, that is for very condensed phase and at the present period of acceleration. We present here a toy model of a complete cosmological scenario in which the main factor responsible for the geometry is a nonlinear magnetic field which produces a FRW homogeneous and isotropic geometry. In this scenario we distinguish four distinct phases: a bouncing period, a radiation era, an acceleration era and a re-bouncing. It has already been shown that in NLED a strong magnetic field can overcome the inevitability of a singular region typical of linear Maxwell theory; on the other extreme situation, that is for very weak magnetic field it can accelerate the expansion. The present model goes one step further: after the acceleration phase the universe re-bounces and enter in a collapse era. This behavior is a manifestation of the invariance under the dual map of the scale factor $a(t) \rightarrow 1/a(t)$, a consequence of the corresponding inverse symmetry of the electromagnetic field ($F \rightarrow 1/F$, where $F \equiv F^{\mu\nu} F_{\mu\nu}$) of the NLED theory presented here. Such sequence collapse-bouncing-expansion-acceleration-re-bouncing-collapse constitutes a basic unitary element for the structure of the universe that can be repeated indefinitely yielding what we call a Cyclic Magnetic Universe.

I. INTRODUCTION

In the last years there has been increasing of interest on the cosmological effects induced by Nonlinear Electrodynamics (NLED) [1]. The main reason for this is related to the drastic modification NLED provokes in the behavior of the cosmological geometry in respect to two of the most important questions of standard cosmology, that is, the initial singularity and the acceleration of the scale factor. Indeed, NLED provides worthwhile alternatives to solve these two problems in a unified way, that is without invoking different mechanisms for each one of them separately. Such economy of hypotheses is certainly welcome. The partial analysis of each one of these problems was initiated in [1, 2]. Here we will present a new cosmological model, that unifies both descriptions.

The general form for the dynamics of the electromagnetic field, compatible with covariance and gauge conservation principles reduces to $L = L(F)$, where $F \equiv F^{\mu\nu} F_{\mu\nu}$. We do not consider here the other invariant $G \equiv F^{\mu\nu} F_{\mu\nu}^*$ constructed with the dual, since its practical importance disappears in cosmological framework once in our scenario the average of the electric field vanishes in a magnetic universe as we shall see in the next sections. Thus, the Lagrangian appears as a regular function that can be developed as positive or negative powers of the invariant F . Positive powers dominate the dynamics of the gravitational field in the neighborhood of its moment of extremely high curvatures [1]. Negative powers control the other extreme, that is, in the case of very weak electromagnetic fields [2]. In this case as it was pointed out previously it modifies the evolution of

the cosmic geometry for large values of the scale factor, inducing the phenomenon of acceleration of the universe. The arguments presented in [4] make it worth considering that only the averaged magnetic field survives in a FRW spatially homogeneous and isotropic geometry. Such configuration of pure averaged magnetic field combined with the dynamic equations of General Relativity received the generic name of Magnetic Universe [3].

The most remarkable property of a Magnetic Universe configuration is the fact that from the energy conservation law it follows that the dependence on time of the magnetic field $H(t)$ is the same irrespective of the specific form of the Lagrangian. This property allows us to obtain the dependence of the magnetic field on the scale factor $a(t)$, without knowing the particular form of the Lagrangian $L(F)$. Indeed, as we will show later on, from the energy-momentum conservation law it follows that $H = H_0 a^{-2}$. This dependence is responsible for the property which states that strong magnetic fields dominates the geometry for small values of the scale factor; on the other hand, weak fields determines the evolution of the geometry for latter eras when the radius is big enough to excite these terms.

In order to combine both effects, here we will analyze a toy model. The symmetric behavior of the magnetic field in both extremes – that is for very strong and very weak regimes – allows the appearance of a repetitive configuration of the kind exhibited by an eternal cyclic universe.

Negative power of the field in the Lagrangian of the gravitational field was used in [5] attempting to explain the acceleration of the scale factor of the universe by modification of the dynamics of the gravitational field

by adding to the Einstein-Hilbert action a term that depends on negative power of the curvature, that is

$$S = \frac{M_{\text{Pl}}^2}{2} \int \sqrt{-g} \left(R - \frac{\alpha^4}{R} \right) d^4x,$$

This modification introduced an idea that is worth to be generalized: the dynamics should be invariant with respect to the inverse symmetry transformation. In other words, if \mathbb{X} represents the invariant used to construct a Lagrangian for a given field, the Action should be invariant under the map $\mathbb{X} \rightarrow 1/\mathbb{X}$. Since the Electrodynamics is the paradigm of field theory, one should start the exam of such a principle into the realm of this theory. In other words we will deal here with a new symmetry between strong and weak electromagnetic field. In [2], a model assuming this idea was presented and its cosmological consequences analyzed. In this model, the action for the electromagnetic field was modified by the addition of a new term, namely

$$S = \int \sqrt{-g} \left(-\frac{F}{4} + \frac{\gamma}{F} \right) d^4x. \quad (1)$$

This action yields an accelerated expansion phase for the evolution of the universe, and correctly describes the electric field of an isolated charge for a sufficiently small value of parameter γ . The acceleration becomes a consequence of the properties of this dynamics for the situation in which the field is weak.

In another cosmological context, in the strong regime, it has been pointed out in the literature [1] that NLED can produce a bouncing, altering another important issue in Cosmology: the singularity problem. In this article we would like to combine both effects improving the action given in Eqn.(1) to discuss the consequences of NLED for both, weak and strong fields.

It is a well-known fact that under certain assumptions, the standard cosmological model unavoidably leads to a singular behavior of the curvature invariants in what has been termed the Big Bang. This is a highly distressing state of affairs, because in the presence of a singularity we are obliged to abandon the rational description of Nature. It is possible that a complete quantum cosmology could describe the state of affairs in a very different and more complete way. For the time being, while such complete quantum theory is not yet known, one should attempt to explore alternatives that are allowed and that provide some sort of phenomenological consequences of a more profound theory.

It is tempting then to investigate how NLED can give origin to an unified scenario that not only accelerates the universe for weak fields (latter cosmological era) but that is also capable of avoiding an initial singularity as a consequence of its properties in the strong regime.

Scenarios that avoid an initial singularity have been intensely studied over the years. As an example of some latest realizations we can mention the pre-big-bang universe [7] and the ekpyrotic universe [6]. While these

models are based on deep modifications on conventional physics, that are extremely difficult to be observed, the model we present here relies instead on the electromagnetic field. The new ingredient that we introduce concerns the dynamics that is rather different from that of Maxwell in distinct regimes. Specifically, the Lagrangian we will work with is given by

$$L_T = \alpha^2 F^2 - \frac{1}{4} F - \frac{\mu^2}{F} + \frac{\beta^2}{F^2}. \quad (2)$$

The dimensional constants α, β and μ are to be determined by observation. Thus the complete dynamics of electromagnetic and gravitational fields are governed by Einstein equations plus L_T .

We shall see that in Friedmann-Robertson-Walker (FRW) geometry we can distinguish four typical eras which generate a basic unity of the cosmos (BUC) that repeat indefinitely.

The whole cosmological scenario is controlled by the energy density ρ and the pressure p of the magnetic field. Each era of the BUC is associated with a specific term of the Lagrangian. As we shall see the conservation of the energy-momentum tensor implies that the field dependence on the scale factor yields that the invariant F is proportional to a^{-4} . This dependence is responsible by the different dominance of each term of the Lagrangian in different phases. The first term $\alpha^2 F^2$ dominates in very early epochs allowing a bouncing to avoid the presence of a singularity. Let us call this the **bouncing era**. The second term is the Maxwell linear action which dominates in the **radiation era**. The inverse term μ^2/F dominates in the **acceleration era**. Finally the last term β^2/F^2 is responsible for a **re-bouncing**. Thus each BUC can be described in the following way:

- The bouncing era: There exists a collapsing phase that attains a minimum value for the scale factor $a_B(t)$;
- The radiation era: after the bouncing, $\rho + 3p$ changes the sign; the universe stops its acceleration and start expanding with $\ddot{a} < 0$;
- The acceleration era: when the $1/F$ factor dominates the universe enters an accelerated regime;
- The re-bouncing era: when the term $1/F^2$ dominates, the acceleration changes the sign and starts a phase in which $\ddot{a} < 0$ once more; the scale factor attains a maximum and re-bounces

The universe starts a collapsing phase entering a new bouncing era. This unity of four stages, the BUC, constitutes an eternal cyclic configuration that repeats itself indefinitely.

The plan of the article is as follows. In section II we review the Tolman process of average in order to conciliate the energy distribution of the electromagnetic field with a spatially isotropic geometry. Section III presents the

notion of the Magnetic Universe and its generic features concerning the dynamics of electromagnetic field generated by a Lagrangian $L = L(F)$. Section IV presents the conditions of bouncing and acceleration of a FRW universe in terms of properties to be satisfied by L . In section V we introduce the notion of inverse symmetry of the electromagnetic field in a cosmological context. This principle is used to complete the form of the Lagrangian that guides the combined dynamics of the unique long-range fields yielding a spatially homogeneous and isotropic nonsingular universe. In sections VI and VII we present a complete scenario consisting of the four eras: a bouncing, an expansion with negative acceleration, an accelerated phase and a re-bouncing. We end with some comments on the form of the scale factor and future developments. In appendix we present the compatibility of our Lagrangian with standard Coulomb law and the modifications induced on causal properties of nonlinear electrodynamics.

II. THE AVERAGE PROCEDURE AND THE FLUID REPRESENTATION

The effects of a nonlinear electromagnetic theory in a cosmological setting have been studied in several articles [9].

Given a generic gauge-independent Lagrangian $L = L(F)$, written in terms of the invariant $F \equiv F_{\mu\nu}F^{\mu\nu}$ it follows that the associated energy-momentum tensor, defined by

$$T_{\mu\nu} = \frac{2}{\sqrt{-\gamma}} \frac{\delta L \sqrt{-\gamma}}{\delta \gamma^{\mu\nu}}, \quad (3)$$

reduces to

$$T_{\mu\nu} = -4 L_F F_\mu{}^\alpha F_{\alpha\nu} - L g_{\mu\nu}. \quad (4)$$

In the standard cosmological scenario the metric structure of space-time is provided by the FLRW geometry. For compatibility with the cosmological framework, that is, in order that an electromagnetic field can generate a homogeneous and isotropic geometry an average procedure must be used. We define the volumetric spatial average of a quantity X at the time t by

$$\overline{X} \equiv \lim_{V \rightarrow V_0} \frac{1}{V} \int X \sqrt{-g} d^3x, \quad (5)$$

where $V = \int \sqrt{-g} d^3x$ and V_0 is a sufficiently large time-dependent three-volume. In this notation, for the electromagnetic field to act as a source for the FLRW model we need to impose that

$$\overline{E_i} = 0, \quad \overline{H_i} = 0, \quad \overline{E_i H_j} = 0, \quad (6)$$

$$\overline{E_i E_j} = -\frac{1}{3} E^2 g_{ij}, \quad \overline{H_i H_j} = -\frac{1}{3} H^2 g_{ij}. \quad (7)$$

With these conditions, the energy-momentum tensor of the EM field associated to $L = L(F)$ can be written as that of a perfect fluid,

$$T_{\mu\nu} = (\rho + p)v_\mu v_\nu - p g_{\mu\nu}, \quad (8)$$

where

$$\begin{aligned} \rho &= -L - 4L_F E^2, \\ p &= L - \frac{4}{3} (2H^2 - E^2) L_F, \end{aligned} \quad (9)$$

and $L_F \equiv dL/dF$.

III. MAGNETIC UNIVERSE

A particularly interesting case occurs when only the average of the magnetic part does not vanishes and $E^2 = 0$. Such situation has been investigated in the cosmological framework yielding what has been called *magnetic universe*. This should be a real possibility in the case of cosmology, since in the early universe the electric field is screened by the charged primordial plasma, while the magnetic field lines are frozen [4]. In spite of this fact, in [2] some attention was devoted to the mathematically interesting case in which $E^2 = \sigma^2 H^2 \neq 0$.

An interesting feature of such magnetic universe comes from the fact that it can be associated with a four-component non-interacting perfect fluid. Let us give a brief proof of the statement that in the cosmological context the energy-content that follows from this theory can be described in terms of a perfect fluid. We work with the standard form of the FLRW geometry in Gaussian coordinates provided by (we limit the present analysis to the Euclidean section)

$$ds^2 = dt^2 - a(t)^2 (dr^2 + r^2 d\Omega^2). \quad (10)$$

The expansion factor, θ defined as the divergence of the fluid velocity reduces, in the present case, to the derivative of logarithm of the scale factor

$$\theta \equiv v^\mu_{;\mu} = 3 \frac{\dot{a}}{a} \quad (11)$$

The conservation of the energy-momentum tensor projected in the direction of the co-moving velocity $v^\mu = \delta^\mu_0$ yields

$$\dot{\rho} + (\rho + p)\theta = 0 \quad (12)$$

Using Lagrangian L_T in the case of the magnetic universe yields for the density of energy and pressure given in equations (9):

$$\rho = -\alpha^2 F^2 + \frac{1}{4} F + \frac{\mu^2}{F} - \frac{\beta^2}{F^2} \quad (13)$$

$$p = -\frac{5\alpha^2}{3} F^2 + \frac{1}{12} F - \frac{7\mu^2}{3} \frac{1}{F} + \frac{11\beta^2}{3} \frac{1}{F^2} \quad (14)$$

where

$$F = 2H^2 \quad (15)$$

Substituting these values in the conservation law, it follows

$$L_F \left[(H^2)' + 4H^2 \frac{\dot{a}}{a} \right] = 0. \quad (16)$$

where $L_F \equiv \partial L / \partial F$.

The important result that follows from this equation is that the dependence on the specific form of the Lagrangian appears as a multiplicative factor. This property shows that any Lagrangian $L(F)$ yields the same dependence of the field on the scale factor irrespective of the particular form of the Lagrangian. Indeed, equation (16) yields

$$H = H_0 a^{-2}. \quad (17)$$

This property implies that for each power F^k it is possible to associate a specific fluid configuration with density of energy ρ_k and pressure p_k in such a way that the corresponding equation of state is given by

$$p_k = \left(\frac{4k}{3} - 1 \right) \rho_k. \quad (18)$$

We restrict our analysis in the present paper to the theory provided by a toy-model described by the Lagrangian

$$\begin{aligned} L_T &= L_1 + L_2 + L_3 + L_4 \\ &= \alpha^2 F^2 - \frac{1}{4} F - \frac{\mu^2}{F} + \frac{\beta^2}{F^2} \end{aligned} \quad (19)$$

where α, β, μ are parameters characterizing a concrete specific model. For latter use we present the corresponding many-fluid component associated to Lagrangian L_T . We set for the total density and pressure $\rho_T = \sum \rho_i$ and $p_T = \sum p_i$ where

$$\begin{aligned} \rho_1 &= -\alpha^2 F^2, \quad p_1 = \frac{5}{3} \rho_1 \\ \rho_2 &= \frac{1}{4} F, \quad p_2 = \frac{1}{3} \rho_2 \\ \rho_3 &= \frac{\mu^2}{F}, \quad p_3 = -\frac{7}{3} \rho_3 \\ \rho_4 &= -\frac{\beta^2}{F^2}, \quad p_4 = -\frac{11}{3} \rho_4. \end{aligned} \quad (20)$$

Or, using the dependence of the field on the scale factor equation (17),

$$\begin{aligned} \rho_1 &= -4\alpha^2 H_0^4 \frac{1}{a^8} \\ \rho_2 &= \frac{H_0}{2} \frac{1}{a^4} \\ \rho_3 &= \frac{\mu^2}{2H_0^2} a^4 \\ \rho_4 &= -\frac{\beta^2}{4H_0^4} a^8. \end{aligned} \quad (21)$$

Let us point out a remarkable property of the combined system of this NLED generated by L_T and Friedman equations of cosmological evolution. A simple look into the above expressions for the values of the density of energy exhibits what could be a possible difficulty of this system in two extreme situations, that is, when F^2 and $1/F^2$ terms dominate, since if the radius of the universe can attain arbitrary small and/or arbitrary big values, then one should face the question regarding the positivity of its energy content. However, as we shall show in the next sections, the combined system of equations of the cosmic metric and the magnetic field described by General Relativity and NLED, are such that a beautiful conspiracy occurs in such a way that the negative contributions for the energy density that came from terms L_1 and L_4 never overcomes the positive terms that come from L_2 and L_3 . Before arriving at the undesirable values where the density of energy could attain negative values, the universe bounces (for very large values of the field) and re-bounces (in the other extreme, that is, for very small values) to precisely avoid this difficulty. This occurs at the limit value $\rho_B = \rho_{RB} = 0$, as follows from equation

$$\rho = \frac{\theta^2}{3}. \quad (22)$$

We emphasize that this is not an extra condition imposed by hand but a direct consequence of the dynamics described by L_T . Indeed, at early stages of the expansion phase the dynamics is controlled by the approximation Lagrangian $L_T \approx L_{1,2} = L_1 + L_2$. Then

$$\rho = \frac{F}{4} (1 - 4\alpha^2 F).$$

Using the conservation law (12) we conclude that the density of energy will be always positive since there exists a minimum value of the scale factor given by $a_{min}^4 = 8\alpha^2 H_0^2$. A similar conspiracy occurs in the other extreme where we approximate $L_T \approx L_{2,3} = L_2 + L_3$, which shows that the density remains positive definite, since $a(t)$ remains bounded, attaining a maximum in the moment the universe makes a re-bounce. These extrema occurs precisely at the points where the total density vanishes. Let us now turn to the generic conditions needed for the universe to have a bounce and a phase of accelerated expansion.

IV. CONDITIONS FOR BOUNCING AND ACCELERATION

1. Acceleration

From Einstein's equations, the acceleration of the universe is related to its matter content by

$$3\frac{\ddot{a}}{a} = -\frac{1}{2}(\rho + 3p). \quad (23)$$

In order to have an accelerated universe, matter must satisfy the constraint $(\rho + 3p) < 0$. In terms of the quantities defined in Eqn. (9),

$$\rho + 3p = 2(L - 4H^2 L_F). \quad (24)$$

Hence the constraint $(\rho + 3p) < 0$ translates into

$$L_F > \frac{L}{4H^2}. \quad (25)$$

It follows that any nonlinear electromagnetic theory that satisfies this inequality yields accelerated expansion. In our present model it follows that terms L_2 and L_4 produce negative acceleration and L_1 and L_3 yield inflationary regimes ($\ddot{a} > 0$).

For latter uses we write the value of $\rho + 3p$ for the case of Lagrangian L_T :

$$\rho + 3p = -6\alpha^2 F^2 + \frac{F}{2} - \frac{6\mu^2}{F} + \frac{10\beta^2}{F^2}.$$

2. Bouncing

In order to analyze the conditions for a bouncing it is convenient to re-write the equation of acceleration using explicitly the expansion factor θ , which is called the Raychaudhuri equation:

$$\dot{\theta} + \frac{1}{3}\theta^2 = -\frac{1}{2}(\rho + 3p) \quad (26)$$

Thus besides condition (25) for the existence of an acceleration a bounce needs further restrictions on $a(t)$. Indeed, the existence of a minimum (or a maximum) for the scale factor implies that at the bouncing point B the inequality $(\rho_B + 3p_B) < 0$ (or, respectively, $(\rho_B + 3p_B) > 0$) must be satisfied. Note that at any extremum (maximum or minimum) of the scale factor the density of energy vanishes. This is a direct consequence of the first integral of Friedman equation which, in the Euclidean case, reduces to equation (22).

V. DUALITY ON THE MAGNETIC UNIVERSE AS A CONSEQUENCE OF THE INVERSE SYMMETRY

The cosmological scenario that is presented here deals with a cyclic FRW geometry which has a symmetric behavior for small and big values of the scale factor. This scenario is possible because the behavior of its energy content at high energy is the same as it has in its weak regime. This is precisely the case of the magnetic universe that we are dealing with here. To obtain a perfect symmetric configuration for our model we will impose a new dynamical principle:

• The inverse symmetry principle:

The NLED theory should be invariant under the inverse map

$$F \rightarrow \tilde{F} = \frac{cte}{F}.$$

For the Lagrangian (19), we have chosen the constant to be $4\mu^2$. This restricts the number of free parameters from three to two, once a direct application of this principle implies that $\beta^2 = 16\alpha^2\mu^4$. This symmetry induces a corresponding one for the geometry. Indeed, the cosmological dynamics is invariant under the associated dual map

$$a(t) \rightarrow \tilde{a}(t) = \frac{H_0}{\sqrt{\mu}} \frac{1}{a} \quad (27)$$

It is precisely this invariance that is at the origin of the cyclic property of this cosmological scenario.

Let us point out that the above map is nothing but a conformal transformation. Indeed, in conformal time, the geometry takes the form

$$ds^2 = a(\eta)^2 (d\eta^2 - dr^2 - r^2 d\Omega^2). \quad (28)$$

Thus making the conformal map

$$\tilde{g}_{\mu\nu} = \omega^2 g_{\mu\nu}$$

where $\omega = \lambda/a^2$, and $\lambda \equiv H_0/\sqrt{\mu}$. Note that although the Lagrangian L_T is not invariant under a conformal transformation, the average procedure used to make compatible the dynamical system of the electromagnetic field and the Friedman equation is invariant. Indeed, we have

$$\tilde{F} = \tilde{g}^{\mu\nu} \tilde{g}^{\alpha\beta} F_{\alpha\mu} F_{\beta\nu} = \frac{4\mu^2}{F}.$$

VI. A COMPLETE SCENARIO

Electromagnetic radiation described by a maxwellian distribution has driven the geometry of the universe for a period. Let us now analyze the modifications introduced by the non linear terms in the cosmic scenario. The simplest way to do this is to combine the previous lagrangian with the dependence of the magnetic field on the scale factor. We set

$$L_T = \alpha^2 F^2 - \frac{1}{4} F - \frac{\mu^2}{F} + \frac{\beta^2}{F^2} \quad (29)$$

where β is related to the other parameters α and μ by the inverse symmetry principle, as displayed above.

A. Potential

It will be more direct to examine the effects of the magnetic universe controlled by the above lagrangian if we undertake a qualitative analysis using an analogy with classical mechanics. Friedman's equation reduces to the set

$$\dot{a}^2 + V(a) = 0 \quad (30)$$

where

$$V(a) = \frac{A}{a^6} - \frac{B}{a^2} - Ca^6 + Da^{10} \quad (31)$$

is a potential that restricts the motion of the localization $a(t)$ of the "particle". The constants in V are given by

$$A = \frac{4\alpha^2 H_0^4}{3}, \quad B = \frac{H_0^2}{6}, \quad C = \frac{\mu^2}{6H_0^2}, \quad D = \frac{4\alpha^2 \mu^4}{3H_0^4},$$

and are positive.

The dependence of the field as $H = H_0/a^2$ implies the existence of four distinct epochs, which we will analyze now.

The derivative dL/dF has three zeros, in which $\rho + p$ vanishes. In the case of pure magnetic universe the value of F is always positive. We distinguish four eras.

B. The four eras of the Magnetic Universe

The dynamics of the universe with matter density given by Eqn.(29) can be obtained qualitatively from the analysis of Einstein's equations. We distinguish four distinct periods according to the dominance of each term of the energy density. The early regime (driven by the F^2 term); the radiation era (where the equation of state $p = 1/3\rho$ controls the expansion); the third accelerated evolution (where the $1/F$ term is the most important one) and finally the last era where the $1/F^2$ dominates and in which the expansion stops, the universe re-bounces and enters in a collapse era.

1. Bouncing era

In the strong field limit the value of the scalar of curvature is small and the volume of the universe attains its minimum, the density of energy and the pressure are dominated by the terms coming from the quadratic lagrangian F^2 and is approximated by the forms

$$\begin{aligned} \rho &\approx \frac{H^2}{2} (1 - 8\alpha^2 H^2) \\ p &\approx \frac{H^2}{6} (1 - 40\alpha^2 H^2) \end{aligned} \quad (32)$$

Using the dependence $H = H_0/a^2$, on equation (30) leads to

$$\dot{a}^2 = \frac{kH_0^2}{6a^2} \left(1 - \frac{8\alpha^2 H_0^2}{a^4} \right) - \epsilon. \quad (33)$$

We remind the reader that we limit our analysis here to the Euclidean section ($\epsilon = 0$). As long as the right-hand side of equation (33) must not be negative it follows that, the scale-factor $a(t)$ cannot be arbitrarily small. Indeed, a solution of (33) is given as

$$a^2 = H_0 \sqrt{\frac{2}{3} (t^2 + 12\alpha^2)}. \quad (34)$$

The radiation period can be achieved from the above equation by setting $\alpha = 0$. As a consequence the average strength of the magnetic field H evolves in time as

$$H^2 = \frac{3}{2} \frac{1}{t^2 + 12\alpha^2}. \quad (35)$$

Note that at $t = 0$ the radius of the universe attains a minimum value at the bounce:

$$a_B^2 = H_0 \sqrt{8\alpha^2}. \quad (36)$$

Therefore, the actual value of a_B depends on H_0 , which - for given α , μ turns out to be the sole free parameter of the model. The energy density ρ reaches its maximum for the value $\rho_B = 1/64\alpha^2$ at the instant $t = t_B$, where

$$t_B = \sqrt{12\alpha^2}. \quad (37)$$

For smaller values of t the energy density decreases, vanishing at $t = 0$, while the pressure becomes negative. Only for very small times $t < \sqrt{4\alpha^2/k}$ the non-linear effects are relevant for cosmological solution of the normalized scale-factor. Indeed, solution (34) fits the standard expression of the Maxwell case at the limit of large times.

2. Radiation era

The standard Maxwellian term dominates in the intermediary regime. Due to the dependence on a^{-2} of the field, this phase is defined by $H^2 \gg H^4$ yielding the approximation

$$\begin{aligned} \rho &\approx \frac{H^2}{2} \\ p &\approx \frac{H^2}{6} \end{aligned} \quad (38)$$

This is the phase dominated by the linear regime of the electromagnetic field. Its properties are the same as described in the standard cosmological model.

3. The accelerated era: weak field drives the cosmological geometry

When the universe becomes larger, negative powers of F dominates and the distribution of energy becomes typical of an accelerated universe, that is:

$$\begin{aligned}\rho &\approx \frac{1}{2} \frac{\mu^2}{H^2} \\ p &\approx \frac{-7}{6} \frac{\mu^2}{H^2}\end{aligned}\quad (39)$$

In the intermediate regime between the radiation and the acceleration regime the energy content is described by the combined form

$$\rho = \frac{H^2}{2} + \frac{\mu^2}{2} \frac{1}{H^2},$$

or, in terms of the scale factor,

$$\rho = \frac{H_0^2}{2} \frac{1}{a^4} + \frac{\mu^2}{2H_0^2} a^4. \quad (40)$$

For small a it is the ordinary radiation term that dominates. The $1/F$ term takes over only after $a = \sqrt{H_0}/\mu$, and would grows without bound afterwards. In fact, the curvature scalar is

$$R = T^\mu_\mu = \rho - 3p = \frac{4\mu^2}{H_0^2} a^4,$$

showing that one could expect a curvature singularity in the future of the universe for which $a \rightarrow \infty$. We shall see, however that the presence of the term $1/F^2$ changes this behavior.

Using this matter density in Eqn.(23) gives

$$3\frac{\ddot{a}}{a} + \frac{H_0^2}{2} \frac{1}{a^4} - \frac{3}{2} \frac{\mu^2}{H_0^2} a^4 = 0.$$

To get a regime of accelerated expansion, we must have

$$\frac{H_0^2}{a^4} - 3\frac{\mu^2}{H_0^2} a^4 < 0,$$

which implies that the universe will accelerate for $a > a_c$, with

$$a_c = \left(\frac{H_0^4}{3\mu^2} \right)^{1/8}.$$

4. Re-Bouncing

For very big values of the scale factor the density of energy can be approximated by

$$\rho \approx \frac{\mu^2}{F} - \frac{\beta^2}{F^2} \quad (41)$$

and we pass from an accelerated regime to a phase in which the acceleration is negative. When the field attains the value $F_{RB} = 16\alpha^2\mu^2$ the universe changes its expansion to a collapse. The scale factor attains its maximum value

$$a_{max}^4 \approx \frac{H_0^2}{8\alpha^2\mu^2}.$$

VII. POSITIVITY OF THE DENSITY OF ENERGY

The total density of energy of the BUC is always positive definite (see equation 22). In the bouncing and in the re-bouncing eras it takes the value $\rho_B = \rho_{RB} = 0$. At these points the density is an extremum. Actually, both points are minimum of the density. This is a direct consequence of equations (12) and (22). Indeed, derivative of (12) at the bouncing and at the re-bouncing yields

$$\ddot{\rho}_B = \frac{3}{2} p_B^2 > 0.$$

Thus there must exists another extremum of ρ which should be a maximum. This is indeed the case since there exists a value on the domain of the evolution of the universe between the two minima such that

$$\rho_c + p_c = 0.$$

At this point we have

$$\ddot{\rho} + \dot{p}_c \theta_c = 0$$

showing that at this point c the density takes its maximum value.

VIII. THE BEHAVIOR OF THE SCALE FACTOR

Let us pause for a while and describe the form of the scale factor as function of time in the four regimes. To simplify such description let us separate in three parts:

Phase A: Bouncing-Radiation

Phase B: Radiation-Acceleration

Phase C: Acceleration-Re-bouncing

characterized respectively by the dynamics controlled by: $L_A = L_1 + L_2$; $L_B = L_2 + L_3$; $L_C = L_3 + L_4$. It is straightforward to obtain an analytical expression for each one of these periods, which can be analytically continued through distinct eras. We obtain for phase A :

$$a_{BR}(t) = \left[\frac{2}{3} H_0^2 \left[(t - t_1)^2 + 12\alpha^2 \right] \right]^{1/4} \quad (42)$$

The inverse symmetric phase C is given by

$$a_{ARB}(t) = \left[\frac{2}{3} \frac{\mu^2}{H_0^2} \left[(t - t_3)^2 + 12\alpha^2 \right] \right]^{-1/4} \quad (43)$$

and for the intermediary phase B , we have:

$$a_{RA}(t) = \left[\frac{H_0^2}{\mu} \frac{1 - \cos \left(Jsn \left(2\sqrt{\frac{2\mu}{3}} (t - t_2), \frac{\sqrt{2}}{2} \right) \right)}{1 - \cos \left(Jsn \left(2\sqrt{\frac{2\mu}{3}} (t - t_2), \frac{\sqrt{2}}{2} \right) \right)} \right]^{1/4} \quad (44)$$

where Jsn is the inverse of a first kind elliptic function, or the Jacobi function *JacobiSN*. In order to express the analytical continuation of the scalar factor through the different eras it is convenient to re-write equation (44) in the inverse form:

$$t - t_2 = \sqrt{\frac{3}{\mu}} \text{EllipticF} \left[\arccos \left(\frac{H_0^2 - \mu a^4}{H_0^2 + \mu a^4} \right), \frac{\sqrt{2}}{2} \right]. \quad (45)$$

In the limit $\mu a^4 \ll H_0^2$ expression (45) becomes

$$t - t_2 = \sqrt{\frac{3}{2H_0^2}} a^2.$$

Now, in the limit $t - t_1 \gg 12\alpha^2$ expression $a_{ARB}(t)$ reduces to

$$a(t) = \left[\frac{2}{3} H_0^2 \left[(t - t_1)^2 \right] \right]^{1/4}$$

corresponding to the radiation era, which is continued in form $a_{RA}(t)$ above by identifying $t_1 = t_2$.

On the opposite case $\mu a^4 \gg H_0^2$ expression (45) goes to

$$t - t_2 = \sqrt{\frac{3}{2\mu}} \left(\pi - \sqrt{H_0^2 \mu} \frac{1}{a^2} \right),$$

that is

$$a(t) = \left[\frac{2}{3} \frac{\mu^2}{H_0^2} \left[(t - t_2)^2 \right] \right]^{-1/4}$$

which is continued analytically to the phase C by the identification $t_3 = t_1 + \pi \sqrt{\frac{3}{2\mu}}$.

IX. CONCLUSION

The simplified toy model presented here displays many regular properties that should be worth of further investigation. In particular, it provides a spatially homogeneous and isotropic FRW geometry which has no Big Bang and

no Big Rip. It describes correctly the radiation era and allows for an accelerated phase without introducing any extra source.

The particular form of the dynamics of the magnetic field is dictated by the inverse principle, which states that the behavior of the field is invariant under the mapping $F \rightarrow \tilde{F} = \frac{4\mu^2}{F}$. This reflects on the symmetric behavior of the geometry by the dual map $a \rightarrow \tilde{a} = H_0/\sqrt{\mu}a$.

The particular form of NLED is based on the principle that shows an intimate relation between strong and weak field configurations. This inverse-symmetry principle reduces the number of arbitrary parameters of the theory and allows for the regular properties of the cosmical model. The universe is a cyclic one, having its main characteristics synthesized in the following steps:

- Step 1: The universe contains a collapsing phase in which the scalar factor attains a minimum value $a_B(t)$;
- Step 2: after the bouncing the universe expands with $\ddot{a} < 0$;
- Step 3: when the $1/F$ factor dominates the universe enters an accelerated regime;
- Step 4: when $1/F^2$ dominates the acceleration changes the sign and starts a phase in which $\ddot{a} < 0$ once more, the scale factor attains a maximum and re-bounces starting a new collapsing phase;
- Step 5: the universe repeats the same behavior passing steps 1, 2, 3 and 4 again and again, indefinitely.

Appendix A: Static and spherically symmetric electromagnetic solution and the asymptotic regime

We have made an analysis of the modification of electrodynamics in a cosmological context. We are not arguing that these effects are more than the response of the universe to local electrodynamics properties. Some decades ago Wheeler and Feynman made a conjecture that local properties of electrodynamics (e.g. the Lienard-Wiechert potential) may just be a consequence of such cosmic response inducing the elimination of advanced fields. However, if one takes these modifications as local change of electrodynamics, we should check consistency of the theory with conventional electromagnetism. We shall restrict ourselves here to the case of the static electric field generated by a point charged particle. For a general nonlinear Lagrangian $L = L(F)$, the EOM for the point charge reduces to

$$r^2 L_F E(r) = \text{const.}$$

In the case of the Lagrangian given in Eqn.(2) we get

$$-\frac{1}{4E^5(r)} (16\alpha^2 E(r)^8 + E(r)^6 - \mu^2 E(r)^2 - \beta^2) = \frac{q}{r^2}. \quad (46)$$

The polynomial in E that follows from this equation cannot be solved exactly, but to study the dependence of E with r we can plot from (46) the function $r = r(E)$.

It also follows from the plot that $E \rightarrow \text{constant}$ for $r \rightarrow \infty$.

By taking derivatives of Eqn.(46) it can be shown that the function $E(r)$ has no extrema [14]. Hence, the modulus of the electric field decreases monotonically with increasing r , from an infinite value at the origin to a constant (nonzero but small) value at infinity. Eqn.(46) then shows that $E_\infty = \text{constant} \neq 0$. This situation is akin to that in the theory defined by the action

$$S = \frac{M_P^2}{2} \int \sqrt{-g} \left(R - \frac{\alpha^4}{R} \right) d^4x.$$

It was shown in [5] that the static and spherically symmetric solution of this theory does not approach Minkowski asymptotically; it tends instead to (anti)-de Sitter space-time. We shall see that a similar situation occurs in the case of NLED.

Regarding the behavior of the field for small values of r , if we compare the term corresponding to Maxwell's case in Eqn.(46) with the other two, we get that for the field to be Maxwell-like there are conditions on the value of the free parameters to be fulfilled, that is:

$$E^2 \ll \frac{1}{16\alpha^2}.$$

$$\mu^2 \ll E^4$$

$$\beta^2 \ll E^6.$$

With the explicit dependence for the field given by $E(r) = q/r^2$, it would be possible to set a value for α in agreement with the observation by

$$\alpha^2 \ll \frac{r_0^4}{16q^2},$$

and analogous expressions for μ and β , where r_0 is a reference value set by the experiment. In the case we fix the value of β by the inverse symmetry, the condition on β reduces to the other two.

5. Asymptotic regime

Let us make an extra comment on the above case of a point charge particle at spatial infinity. The energy-momentum tensor has the form:

$$T_{\mu\nu} = -L\eta_{\mu\nu} - 4L_F F_{\mu\alpha} F^{\alpha\nu} \quad (47)$$

which in the present case $F_{01} = E(r)$ is

$$\begin{aligned} T_0^0 = T_1^1 &= \frac{1}{4E^4} (48\alpha^2 E^8 + 2E^6 - 6\mu^2 E^2 - 5\beta^2) \\ T_2^2 = T_3^3 &= -\frac{1}{4E^4} (16\alpha^2 E^8 + 2E^6 + 2\mu^2 E^2 - \beta^2) \end{aligned} \quad (48)$$

In the asymptotic regime, we can set $E = E_\infty = X$ and, for the energy-momentum tensor one obtains

$$T_0^0 = T_1^1 = T_2^2 = T_3^3 = -\frac{1}{4X^2} (X^3 + 3\mu^2 X + 2\beta^2) \quad (49)$$

which mimics a Λ term.

If we add an extra term in the Lagrangian we could eliminate the residual constant field at infinity. In the case of Maxwell Electrodynamics such ambiguity of choice does not arise due to its linearity. However, for non-linear electromagnetic theory a new possibility occurs which concerns the geometrical structure at infinity. This means that for the non-linear electrodynamics the fact that at infinity the field is a constant does not implies that it vanishes. Such property can be translated in a formal question, that is, what is the asymptotic regime of the geometry of space-time: Minkowski or de Sitter? .

In classical linear Electrodynamics the answer to that question was known and did not pose any ambiguity. No longer so if non-linear electromagnetic field is combined with the equations of general relativity. The possibility of the de Sitter structure must be considered. In theories in which a solution distinct from zero for the equation $L_F = 0$ exists, such a question has to be investigated combined with cosmology. In a recent paper [12] a new look into this question was considered by the exam of a proposed relation of the apparent mass of the graviton and the cosmological constant. We will come back to this question elsewhere.

Appendix B: The fundamental state

A simple look into the equation of motion in NLED shows the existence of a very particular solution such that its energy distribution is the same as the one in the vacuum fundamental state represented by an effective cosmological constant. Indeed, the eom is given by

$$(L_F F^{\mu\nu})_{;\nu} = 0. \quad (50)$$

Consider the particular solution $F = F_0 = \text{constant}$ such that

$$2\alpha^2 F^4 - \frac{F^3}{4} + \mu^2 F - 2\beta^2 = 0.$$

This is the condition that satisfies the equation of motion since L_F vanishes at this value F_0 . In this state the corresponding energy-momentum takes the form

$$T_{\mu\nu} = \Lambda g_{\mu\nu},$$

where

$$\Lambda = -L(F_0) = \frac{1}{F_0^2} \left(\frac{1}{8} F_0^3 + \frac{3\mu^2}{2} F_0 - 2\beta^2 \right).$$

This property is typical of NLED since there is no possibility of the linear Maxwell theory having such particular solution.

Appendix C: Causality

Most[15] of our description of the universe is based on the behavior of light in a gravitational field. In a FLRW scenario the existence of a horizon inhibiting the complete interchange of information between arbitrary parts of the universe associated with the observation of the high degree of isotropy of the CBR generated a causal difficulty: different parts of the universe - in the standard FLRW geometry could not have enough time to homogenize. The inflationary scenario had its appeal precisely for the solution it brought to this problem.

Non linear theories of Electrodynamics presented a completely new look into the causal properties, which we will now overview very briefly.

6. Causal Structure of Nonlinear Electrodynamics

The main lesson we can extract from the analysis of the propagation of the wave front in nonlinear theory is contained in a theorem that can be demonstrated using Hadamard method to deal with the discontinuity of the field and which can be synthesized in a single sentence dealing with the modification of the geometry, generating an effective metric that controls the properties of the space. In order to show this let us make a very short summary of it.

7. The Effective Metric

We decided to give a very short review of the propagation of the electromagnetic waves in the NLED, since it has very peculiar properties that modify the standard description in the linear Maxwellian case. The reader interested in more details can consult [13].

Historically, the first example of the idea of effective metric was presented in 1923 by W. Gordon. In modern language, the wave equation for the propagation of light in a moving non-dispersive medium, with slowly varying refractive index n and 4-velocity u^μ :

$$[\partial_\alpha \partial^\alpha + (n^2 - 1)(u^\alpha \partial_\alpha)^2] F_{\mu\nu} = 0.$$

Taking the geometrical optics limit, the Hamilton-Jacobi equation for light rays can be written as $g^{\mu\nu} k_\mu k_\nu = 0$ (see [13] for details), where

$$g^{\mu\nu} = \eta^{\mu\nu} + (n^2 - 1)u^\mu u^\nu \quad (51)$$

is the effective metric for this problem. It must be noted that only photons in the geometric optics approximation move on geodesics of $g^{\mu\nu}$: the particles that compose the fluid couple instead to the background Minkowskian metric. Let us now study in detail the example of nonlinear electromagnetism. We start with the action

$$S = \int \sqrt{-\gamma} L(F) d^4x, \quad (52)$$

where $F \equiv F^{\mu\nu} F_{\mu\nu}$ and L is an arbitrary function of F . Notice that γ is the determinant of the background metric. which we take in the following to be that of flat spacetime, but the same techniques can be applied when the background is curved. Varying this action w.r.t. the potential W_μ , related to the field by the expression

$$F_{\mu\nu} = W_{\mu;\nu} - W_{\nu;\mu} = W_{\mu,\nu} - W_{\nu,\mu},$$

we obtain the Euler-Lagrange equations of motion (EOM)

$$(L_F F^{\mu\nu})_{;\nu} = 0, \quad (53)$$

where L_F is the derivative $L_F \equiv \partial L / \partial F$. In the particular case of a linear dependence of the Lagrangian with the invariant F we recover Maxwell's equations of motion.

As mentioned in the Introduction, we want to study the behaviour of perturbations of these EOM around a fixed background solution. Instead of using the traditional perturbation method, we shall use a more elegant method set out by Hadamard. In this method, the propagation of low-energy photons are studied by following the evolution of the wave front, through which the field is continuous but its first derivative is not. To be specific, let Σ be the surface of discontinuity defined by the equation

$$\Sigma(x^\mu) = \text{constant}.$$

The discontinuity of a function J through the surface Σ will be represented by $[J]_\Sigma$, and its definition is

$$[J]_\Sigma \equiv \lim_{\delta \rightarrow 0^+} (J|_{\Sigma+\delta} - J|_{\Sigma-\delta}).$$

The discontinuities of the field and its first derivative are given by

$$[F_{\mu\nu}]_\Sigma = 0, \quad [F_{\mu\nu,\lambda}]_\Sigma = f_{\mu\nu} k_\lambda, \quad (54)$$

where the vector k_λ is nothing but the normal to the surface Σ , that is, $k_\lambda = \Sigma_{,\lambda}$, and $f_{\mu\nu}$ represents the discontinuity of the field.

To set the stage for the nonlinear case, let us first discuss the propagation in Maxwell's electrodynamics, for which $L_{FF} = 0$. The EOM then reduces to $F_{;\nu}^{\mu\nu} = 0$, and taking the discontinuity we get

$$f^{\mu\nu} k_\nu = 0. \quad (55)$$

The other Maxwell equation is given by $F_{\mu\nu}^{*;\nu} = 0$ or equivalently,

$$F_{\mu\nu;\lambda} + F_{\nu\lambda;\mu} + F_{\lambda\mu;\nu} = 0. \quad (56)$$

The discontinuity of this equation yields

$$f_{\mu\nu} k_\lambda + f_{\nu\lambda} k_\mu + f_{\lambda\mu} k_\nu = 0. \quad (57)$$

Multiplying this equation by k^λ gives

$$f_{\mu\nu} k^2 + f_{\nu\lambda} k^\lambda k_\mu + f_{\lambda\mu} k^\lambda k_\nu = 0, \quad (58)$$

where $k^2 \equiv k_\mu k_\nu \gamma^{\mu\nu}$. Using the orthogonality condition from Eqn.(55) it follows that

$$f^{\mu\nu} k^2 = 0 \quad (59)$$

Since the tensor associated to the discontinuity cannot vanish (we are assuming that there is a true discontinuity!) we conclude that the surface of discontinuity is null w.r.t. the metric $\gamma^{\mu\nu}$. That is,

$$k_\mu k_\nu \gamma^{\mu\nu} = 0. \quad (60)$$

It follows that $k_{\lambda;\mu} k^\lambda = 0$, and since the vector of discontinuity is a gradient,

$$k_{\mu;\lambda} k^\lambda = 0. \quad (61)$$

This shows that the propagation of discontinuities of the electromagnetic field, in the case of Maxwell's equations (which are linear), is along the null geodesics of the Minkowski background metric.

Let us apply the same technique to the case of a non-linear Lagrangian for the electromagnetic field, given by $L(F)$. Taking the discontinuity of the EOM, Eqn.(53), we get

$$L_F f^{\mu\nu} k_\nu + 2\eta L_{FF} F^{\mu\nu} k_\nu = 0, \quad (62)$$

where we defined the quantity η by $F^{\alpha\beta} f_{\alpha\beta} \equiv \eta$. Note that contrary to the linear case in which the discontinuity tensor $f_{\mu\nu}$ is orthogonal to the propagation vector k^μ , here there is a complicated relation between the vector $f^{\mu\nu} k_\nu$ and quantities dependent on the background field. This is the origin of a more involved expression for the evolution of the discontinuity vector, as we shall see next. Multiplying equation (58) by $F^{\mu\nu}$ we obtain

$$\eta k^2 + F^{\mu\nu} f_{\nu\lambda} k^\lambda k_\mu + F^{\mu\nu} f_{\lambda\mu} k^\lambda k_\nu = 0. \quad (63)$$

Now we substitute in this equation the term $f^{\mu\nu} k_\nu$ from Eqn.(62), and we arrive at the expression

$$L_F \eta k^2 - 2 L_{FF} \eta (F^{\mu\lambda} k_\mu k_\lambda - F^{\lambda\mu} k_\mu k_\lambda), \quad (64)$$

which can be written as $g^{\mu\nu} k_\mu k_\nu = 0$, where

$$g^{\mu\nu} = L_F \gamma^{\mu\nu} - 4 L_{FF} F^{\mu\alpha} F_\alpha{}^\nu. \quad (65)$$

We then conclude that the high-energy photons of a *non-linear* theory of electrodynamics with $L = L(F)$ do not propagate on the null cones of the background metric but on the null cones of an *effective* metric, generated by the self-interaction of the electromagnetic field.

8. Causal properties in the fundamental state

Let us look for the causal structure in the case the electromagnetic field rests on its fundamental state. From the calculation made in the previous chapter the photons propagate in an effective geometry which is given by equation (65). In the case of the inverse symmetry Lagrangian the effective metric tensor takes the form:

$$g_{(eff)}^{\mu\nu} = \left(-\frac{1}{4} + \frac{\mu^8}{F^2}\right) g^{\mu\nu} + \frac{8\mu^8}{F^3} F^{\mu\alpha} F_\alpha{}^\nu. \quad (66)$$

The fundamental state is the particular solution in which

$$F^2 = 4\mu^8,$$

which corresponds to an energy-momentum tensor equivalent to a fluid distribution characterized by the condition $\rho + p = 0$ and generates a deSitter geometry for the background metric $g_{\mu\nu}$ as seen by all forms of matter and energy content - as far as we neglect the gravitational influence of such remaining matter and energy. However, from the above calculation, we conclude that the photons do not propagate in such deSitter space but instead in an effective metric which is provided by the form:

$$g_{(eff)}^{\mu\nu} = \pm \frac{1}{\mu^4} F^{\mu\alpha} F_\alpha{}^\nu. \quad (67)$$

This is a very peculiar and interesting situation that can be described by the following sentence:

- *The fundamental state of the theory described by the inverse symmetric Lagrangian generates a deSitter universe felt by all existing matter with one exception: the photons which follow geodesics in the above anisotropic geometry $g_{(eff)}^{\mu\nu}$.*

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- [14] Note that $E = 0$ is not a solution of Eqn.(46).
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